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Model analysis by the Moiré method

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MODEL ANALYSIS BY THE MOIRE' METHOD

BY

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A

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ABSTRACT

This study presents the use of Moire' fringes for model analysis of deflections and strains. Deflection of a beam, strain in a disk, and an influence diagram for a frame are presented as examples of uses of the Moire' method. The necessary equations for the determination of strain using the Moire' method are also derived and presented in this work.

INTRODUCTION

The Moire' effect is an optical phenomenon which occurs when two arrays of lines or dots are superimposed on one another. When the two arrays are superimposed and one slightly rotated or displaced with respect to the other, alternating light and dark bands normally referred to as fringes, occur. The fringes are usually faint and resemble that of the watered silk effects of Moire' and similar fabrics when two pieces are in contact. Moire' fringes can commonly be seen by looking through two window screens, as the bothersome lines on television, or on some curtain fabrics when hung in such a manner as to be folded.

The fringes occur as a result of, and their characteristics depend on, a relative displacement, angular and/or linear, between the two arrays of lines. Because of this, the fringe characteristics may be used to interpret movements and deformations of bodies. It is the purpose of this study to investigate the properties of Moire' fringes related to model study of structures and strain analysis.

The analysis of structures may be approached from many different experimental or theoretical methods. On simple structures the theoretical or mathematical approaches probably give the quickest results and are usually accurate enough to satisfy the engineer. As the structure becomes more complicated the mathematical approach to the problem becomes more difficult, and the assumptions more critical, leading to more questionable results. In the latter situation the use of a model becomes more advantageous, since with a proper model the true geometry of the structure can be closely approached. There are several accepted methods of model study, among these are the use of strain gages to determine stress at various points and the use of photoelasticity to determine inflection points. Perhaps the most widely used method of model study is based on Maxwell's Law of Reciprocal Displacements. In this case if a structure is given a unit displacement in the direction of one of the redundant reactions, the deflection curve of the structure will represent the influence line for that reaction. The Begg's Deformeter has been the accepted instrument in this connection. It will be shown that the Moire' method provides a quick accurate approach to the measurement of deflections and thus to the determination of influence coordinates.

The measurement of strains using Moire' fringes perhaps would have its greatest appeal to those people concerned with large strains, possibly in the plastic range of a material. It will be shown that there are definite mathematical relationships between the characteristics (spacing and slope) of the fringes and the strain in a model.

The study and use of Moire' fringes in practical problems is relatively recent, dating back only to 1948. The possibilities of the method therefore are not known for fields other than structural mechanics. It can be said however that if there is a case where a relative movement is to be measured, Moire' fringes may prove to be an important tool.

REVIEW OF LITERATURE

The effect with which we are concerned in this study was observed as long ago as 1874 by Lord Rayleigh. Quoting from a book by Guild(1); "In a paper 'On the manufacture and Theory of Diffraction Gratings', he (Rayleigh) wrote: If two photographic copies (of Diffraction gratings) containing the same number of lines to the inch be placed in contact, film to film, in such a manner that the lines are nearly parallel in the gratings, a system of parallel bars develops itself, whose direction bisects the external angle between the directions of the original lines and whose distance increases as the angle of inclination diminishes.....the interval between the bars is evidently half the long diagonal of the rhombus formed by two pairs of consecutive lines, and is expressed by $\frac{a \cos \theta/2}{\sin \theta}$ or $\frac{a}{\theta}$, where a is the interval between the primary lines and θ is the mutual inclination of the two sets. When parallelism is very closely approached the bars become irregular in consequence of the imperfection of the gratings. This phenomenon might be useful as a test." The parallel bars referred to by Lord Rayleigh are now commonly called Moire' fringes.

It appears that there is no further mention in Literature of the fringes or test described by Lord Rayleigh until the late 1940's when a paper was presented by R. Weller and B. M. Shepard(2) in which they discussed some possible uses of the Moire' fringe, among these were the measurements of deflections and strains. Therefore for

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1 - All references are in the Bibliography.

approximately 75 years no practical use was developed for Lord Rayleigh's observation. During this period however, the use of grid systems for the measurement of strains, particularly in the plastic range, was developed. This method consists of placing an orthogonal grid on a model or part to be tested, allowing the part to strain, and then measuring the amount of deformation in the grid. This method works well for large strains, but its use is limited because of the sensitivity of measuring the deformed grid and because there is no way of obtaining an overall picture of strain in the specimen. Also, normally there would be no way to observe the strain at any stage of deformation. The method used in this study essentially is the same as that just described, however the use of the second grid creating Moire' fringes provides a sensitive measuring device for the original grid. Also during the 1940's the method of photoprinting was perfected such that grids could be printed directly on a model. This method of placing a grid on a model has been used by most of the experimenters that have published papers on the subject of Moire' fringes. The photoprinting technique, as described by J. A. Miller,[3] uses a master grid on a negative which can be printed directly on the part to be tested which has previously been treated with a photosensitive coating. The accuracy of the grid is directly related to the accuracy of the master grid. Grids up to approximately 450 lines per inch can be reproduced easily using this method.

Since Moire' fringes depend on the obstruction of light it is necessary to explain the phenomenon from a discussion of light. From the concepts published by Guild[1], the optical phenomenon of Moire' fringes can be explained under conditions suitable for diffraction gratings. In this case if a monochromatic point light source passes through a diffraction grating (a grating which has a spacing between the lines near the wave length of light being used, usually 10,000 to 30,000 lines per inch) the light will spread and break into alternating light and dark bands. If a second grating is placed near the first grating with the same orientation and grid spacing the alternating dark and light bands will remain in the same position but the intensity of light passing the second grating will vary. If the second grating is displaced slightly with respect to the first grating, either translated or rotated, the light and dark bands will have rotated and translated in accordance with the translation or rotation of one grid with respect to the other. Since the light and dark bands are oriented in accordance with the movement of one grid with respect to the other they would provide a means of measuring this relative movement.

Probably a better explanation for this particular study would be the obstruction of light due to shadow effect. In this case if the two grids are composed of equal width black lines separated by clear spaces and the two grids are in contact, or nearly in contact, and originally oriented in the same position no fringes will be seen. If one grid is displaced with respect to the other there will be alternating light and dark bands formed. The dark bands or fringes being due to the mechanical interference of light where one black line of one grid crosses a clear space of the other grid.

Since the conditions under which this study was conducted follows the latter explanation of Moire' fringes, this is the method which will be used in the explanation and derivation of conditions under which Moire' fringes will be obtained.

Because the use of Moire' fringes to make strain and deflection measurements is fairly new, very little information is available on tests which have been performed. A. Vinckier and R. Dechaene[4] used the method to measure plastic strains in steel on several different models. In their technique they used a polished steel plate on which they photoprinted a grid of 200 lines per centimeter. After loading the specimen they photographed the model through a glass plate on which a similar grid had been placed. Their results tended to agree with existing opinions on plastic strains.

Morse, Durelli, and Sciammarella[5] also used the Moire' method to make strain measurements. In this case they photoprinted a rubber disk with 300 lines per inch and took a picture of it, then after loading the disk, re-exposed the film giving them the deformed grid superimposed on the original grid with the resulting Moire' fringes. Their results were very accurate leading them to the conclusion that the main source of error in the test was the flattening of the ends of the disk which leads to an error in the theory, not in the experimental results.

It would appear from literature that the Moire' method of measuring strains leads to very good results if there is a large amount of strain involved or if the grid is sufficiently fine to yield enough fringes for good measurements.

The use of Moire' fringes to determine deflections particularly for influence diagrams on structures also seems to yield good results. Durelli and Daniel[6] determined influence lines for several different types of frames. In their tests they used two ways of applying grids to the models. In one case they used transparent sheets bearing prints of 60 lines per inch which were cemented to the model and in the other case they used photoprinted grids up to 300 lines per inch on the model.

DERIVATION OF STRAIN EQUATIONS

In order to make strain calculations from Moire' fringes the relationships between the fringes and the relative movement between the two grids must be developed. Figure 1 represents the case where one grid has been rotated and strained with respect to the other grid.

In developing the subsequent relationships, the original set of parallel lines will be referred to as the master grid and the spacing or pitch of this grid will be designated as p. The parallel lines on the model will be referred to as the model grid and the spacing or pitch of these lines will be designated as p'. It is to be understood that in this problem p is equal to p' before testing.

Further designations are as follows:

1) The angle θ represents the angle between the model grid and the master grid at any point and is measured from the master grid.

2) The angle ϕ represents the angle between the master grid and the fringes measured at any point from the master grid in the same direction as θ.

3) δ represents the perpendicular distance between any two fringes measured at any point.

By reducing the pitch of figure 1 and increasing the width of the lines the alternating light and dark fringes can be seen as in figure 2. As the pitch of the lines is reduced the fringes would appear as more regular bands.

It is apparent from figure 2 that the dark fringes appear where the dark lines of one grid cover the light spaces of the other grid. This mechanical interference or shadow effect can be used to measure the deflection of one grid with respect to the other as will be seen later. In the following derivations it will be convenient to use the light fringes because of the geometry of the figure. The same relationships hold for the dark fringes however since they are at the same inclination and halfway between the light fringes.

Nominal strain is defined as the change in length of a segment over a set gage length. In this case the gage length will be the master grid pitch.

The strain perpendicular to the master grid would be given by

$$\epsilon_{NOM.} = \frac{p' - p}{p} = \frac{p'}{p} - 1$$

True strain could also be measured by

$$\epsilon_{TRUE} = \frac{p' - p}{p'} = 1 - \frac{p}{p'}$$

It is then necessary to get relationships between p' and the characteristics of the fringes. Referring to figure 1.

$$\overline{AB} = \frac{p'}{\cos[\phi - (\pi/2 + \theta)]} = \frac{p'}{\sin(\phi - \theta)} \qquad (1)$$

and

$$\overline{AB} = \frac{p}{\cos(\phi - \pi/2)} = \frac{p}{\sin \phi} \qquad (2)$$

therefore

$$\frac{P'}{\sin[\phi - \theta]} = \frac{P}{\sin \phi} \tag{3}$$

$$P' = \frac{P \sin[\phi - \theta]}{\sin \phi} \tag{4}$$

Equation (4) relates the model grid, master grid, angle of inclination of the fringes and the rotation of the model grid. It is necessary to eliminate the rotation θ from this equation since this is a term which cannot be measured or would not normally be known and all other values are either known or can be measured from a photograph. To eliminate this term the distance between the fringes δ will be used.

Using the distance a from figure 1.

$$a = \frac{P}{\sin \theta} \tag{5}$$

$$\delta = a \cos[\phi - (\pi/2 + \theta)] = a \sin[\phi - \theta] \tag{6}$$

$$\delta = \frac{P \sin[\phi - \theta]}{\sin \theta} \tag{7}$$

From equation 4

$$P' = \frac{P \sin[\phi - \theta]}{\sin \phi} \tag{4}$$

$$\sin[\phi - \theta] = \frac{p' \sin\phi}{p} \tag{8}$$

Therefore

$$\delta = \frac{p' \sin\phi}{\sin\theta} \tag{9}$$

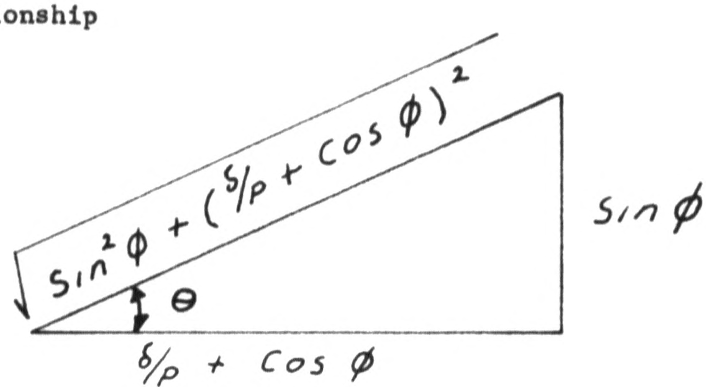
from equation (7)

$$\frac{\delta}{p}\sin\theta = \sin[\phi - \theta] = \sin\phi\cos\theta - \sin\theta\cos\phi$$

$$\frac{\delta}{p}\tan\theta = \sin\phi - \cos\phi\tan\theta$$

$$\tan\theta = \frac{\sin\phi}{\delta/p + \cos\phi} \tag{10}$$

Using the relationship



$$\sin\theta = \frac{\sin\phi}{\sqrt{\sin^2\phi + (\delta/p + \cos\phi)^2}} \tag{11}$$

From equation (9)

$$\delta = \frac{p' \sin\phi}{\sin\theta} = p'\sqrt{\sin^2\phi + (\delta/p + \cos\phi)^2} \tag{12}$$

and

$$p' = \frac{\delta}{\sqrt{\sin^2\phi + (\delta/p + \cos\phi)^2}} \tag{13}$$

$$p' = \frac{\delta}{\sqrt{1 + (\delta/p)^2 + 2\,\delta/p\,\cos\phi}}$$

$$p' = \frac{\delta P}{\sqrt{P^2 + \delta^2 + 2\delta P\cos\phi}}$$

$$\frac{p'}{P} = \frac{\delta}{\sqrt{P^2 + \delta^2 + 2\delta P\cos\phi}} \tag{14}$$

Equation (14) is in terms of values which can be measured from a photograph and the nominal strain would then be

$$\epsilon_{NOM.} = \frac{\delta}{\sqrt{P^2 + \delta^2 + 2\delta P\cos\phi}} - 1 \tag{15}$$

Although all values from equation (15) can be measured from a photograph of the fringes, it might be difficult to measure the perpendicular distance between the fringes. To avoid this situation measurements can be made of the distance between the fringes in a direction parallel to the master grid and perpendicular to the master grid. Designating the direction parallel to the master grid as (s) and the direction perpendicular to the grid as (r) we can calculate δ_r and δ_s from figure 3.

$$\delta = \delta_r \cos(\pi - \phi) = \delta_r(-\cos\phi)$$

To avoid the negative sign on a measured distance, let

$$\delta = \delta_r |\cos\phi| \tag{16}$$

and

$$\delta = \delta_s \sin(\pi - \phi) = \delta_s \sin\phi \tag{17}$$

Substituting equation (16) into equation (14)

$$\frac{P'}{P} = \frac{\delta_r |\cos\phi|}{\sqrt{P^2 + \delta_r^2 \cos^2\phi + 2\,\delta_r\, P|\cos\phi|\cos\phi}} \tag{18}$$

$$\frac{P'}{P} = \frac{\delta_r |\cos\phi|}{\frac{\cos\phi}{\cos\phi}\sqrt{P^2 + \delta_r^2\cos^2\phi + 2\,\delta_r P|\cos\phi|\cos\phi}}$$

$$\frac{P'}{P} = \frac{\delta_r \frac{|\cos\phi|}{\cos\phi}}{\sqrt{\frac{P^2}{\cos^2\phi} + \delta_r^2 + 2P\delta_r \frac{|\cos\phi|}{\cos\phi}}} \tag{19}$$

The previous equations were derived for an obtuse angle ϕ , however by using the following constant, the same results may be obtained for any angle ϕ .

$$\frac{|\cos\phi|}{\cos\phi} = c = \pm 1 \tag{20}$$

If $0 \leq \phi < \pi/2$, $c = +1$

$\pi/2 < \phi \leq \pi$, $c = -1$

$$\frac{P'}{P} = \frac{c\,\delta_r}{\sqrt{\frac{P^2}{\cos^2\phi} + \delta_r^2 + 2PC\delta_r}} \tag{21}$$

Now substituting equation (17) into equation (14)

$$\frac{P'}{P} = \frac{\delta_s \sin\phi}{\sqrt{P^2 + \delta_s^2 \sin^2\phi + 2\delta_s \sin\phi}\cos\phi} \tag{22}$$

using the identity

$$2\sin\phi\cos\phi = \sin 2\phi$$

$$\frac{P'}{P} = \frac{\delta_s \sin\phi}{\sqrt{P^2 + \delta_s^2 \sin^2\phi + \delta_s \sin 2\phi}} \tag{23}$$

The nominal strain, using the distance between the fringes in the coordinate directions, then becomes

$$\epsilon_{NOM.} = \frac{c\,\delta_r}{\sqrt{\frac{P^2}{\cos^2\phi} + \delta_r^2 + 2PC\,\delta_r}} - 1 \tag{24}$$

$$\epsilon_{NOM} = \frac{\delta_s \sin\phi}{\sqrt{P^2 + \delta_s^2 \sin^2\phi + \delta_s \sin 2\phi}} - 1 \tag{25}$$

The shear strain and rigid body rotation may be determined from the relationships derived from figure 4.

Designating shear strain by γ and assigning a positive sign to counterclockwise angles.

$$\gamma = \theta_x - \theta_y \tag{26}$$

and designating the rigid body rotation as β.

$$\beta = \frac{\theta_x + \theta_y}{2} \tag{27}$$

An approximate equation for the nominal strain can be obtained when the angle ϕ approaches zero or 180 degrees. In this case from equation 24.

$$\epsilon = \frac{\pm \delta_r}{\sqrt{p^2 \pm 2p\delta_r + \delta_r^2}} - 1 \tag{28}$$

$$\epsilon = \frac{\pm \delta_r}{p \pm \delta_r} - 1 = \frac{-p}{p \pm \delta_r} \tag{29}$$

Furthermore if p is much smaller than δ_r , from equation (29)

$$\epsilon = \pm \frac{p}{\delta_r} \tag{30}$$

The notation used in this section is the same as that used by Morse, Durelli, and Sciammeralla[5].

TEST PROCEDURE

Three tests were conducted to demonstrate the use of Moire' fringes for the measurement of strains and deflections. The three tests were: the measurement of the deflection of a beam, the measurement of strain in a disk, and the measurement of the deformation of a frame.

1) Beam deflection test

A 10 inch long, 1 inch deep, and 1/4 inch thick beam was machined from plexiglass and a horizontal grid of 60 lines per inch was placed on the beam. The method used in placing the grid on the beam was by using commercially available clear plastic sheets with 60 lines per inch printed on them. These sheets have a pressure sensitive cement backing and are easily applied to a model. The beam was placed on two simple supports in the loading frame associated with the polariscope in the Mechanics Department Laboratory. A second grid was held against the beam and oriented so that no fringes were visible. The beam was then loaded with a concentrated load at its mid point. The results of this test are shown in figure 5. Each light fringe represents 1/60 of an inch of deflection from the proceeding light fringe. Knowing that at the supports the deflection is zero, the deflection at any point in the beam can be determined. Measuring along the center line of the beam where the strain would be zero, a deflection curve was obtained as shown in figure 6. From the maximum deflection at the center of the span, the modulus of elasticity of the plexiglas was obtained.

In this case a maximum deflection of 8/60 of an inch or 0.133 inches was obtained with a load of 58 pounds at the center of the beam. This gives a modulus of elasticity of 435,000 pounds per square inch which compares with the range of values given for the modulus of elasticity for plexiglas. Using this value of the modulus of elasticity a theoretical deflection curve for the entire beam was also plotted on figure 6. It can be seen that the two curves very closely coincide.

2) Strain in a disk

A grid of 200 lines per inch was machined on a 1/2 inch thick piece of plexiglas. The method used here to place the grid on the plexiglas was to cut very shallow grooves 0.005 inches apart and then to spread ink on the surface thus filling the grooves with a black material and leaving the spaces between them clear. The grooves were machined in the Mechanical Engineering Laboratory on a shaper. After the grooves were cut and blacked in, a 4 inch diameter disk was machined from the plexiglas. The disk was placed in the Tinius Olsen hydraulic testing machine in the Mechanics Department Laboratory. An initial load of 1000 pounds was applied to the disk and then a second piece of plexiglas with a similar grid was placed against the disk and oriented such that the fringe pattern would be symmetrical. This loading procedure was used because it was found to be difficult to align the grids perfectly before loading. The disk was then stressed with a 4000 pound load.

With the grid perpendicular to the load a fringe pattern as shown in figure 7 was obtained. The strain was calculated from this fringe pattern along the diameter in the direction of the load. The strain was obtained by using the approximate equation (30). The use of this equation is justified since the strains are small and the angle the fringe makes with the horizontal is 0 degrees. A graph was plotted of accumulated fringe distance (from the top of the disk) against an arbitrary fringe number. The inverse slope of this curve gives the distance between the fringes at any point. The curve was plotted for a little less than half of the disk because it became difficult to distinguish the center of the fringes near the center of the disk. Using the slope of this curve and the approximate equation for strain, a curve of vertical strain versus the distance from the top of the disk was plotted in figure 9. This curve agreed closely with the theoretical curve (7) using the previously determined modulus of elasticity of 435,000 psi and assuming a value for Poisson's ration of 0.25. The value of Poisson's ratio is not known and would have to be determined experimentally as it is not generally listed in tables with the other properties of materials. It was thought that the value could be obtained from a picture of the fringes with the grid oriented in a vertical direction which would give the strain perpendicular to the load. This picture was obtained, as shown in figure 10, but due to the small amount of strain in the horizontal direction no measurements were attempted from the picture. The value of Poisson's ratio for non-porous materials usually varies between 0.25 and 0.50(8) with 0.50 being the maximum value possible. Since a stress-strain curve for plexiglas shows it to be a rather brittle material and since brittle materials tend to have a lower value for Poisson's ratio, a value of 0.25 was selected. Because the stress in the direction perpendicular to the load is small in comparison to the stress parallel to the load, little error would be introduced by changing Poisson's ratio slightly.

3) Influence Diagram for a frame

A frame as shown in figure 11 was machined from a 1/8 inch thick sheet of plexiglas and a grid of 60 lines per inch was placed on the frame in the same manner as in the beam test.

A flat plate with approximately the same outside dimensions as the frame was also cut from the same sheet of plexiglas and a similar grid was placed on it. Holes were drilled in the plate and in the frame such that no fringes would appear when the frame was pinned to the plate. A second hole was drilled for the right leg such that to pin the leg the end would have to be moved horizontally approximately 0.3 of an inch. It was found later that this distance was nearer 0.26 of an inch due to the looseness of the fit between the pin and the hole. It is not necessary to know this dimension to draw the influence diagram. This procedure was used with the grids placed on the plate and frame first vertically and then horizontally with the results shown in figures12 and 13. From Maxwell's Law of Reciprocal Displacements an influence diagram was drawn for the horizontal reaction at the right support. This was compared against the theoretical influence diagram obtained by the dummy load method. The two diagrams as shown in figure 14 closely coincide except at the joints and this was to be expected since the theoretical calculations do not take into account the added stiffness at a joint.

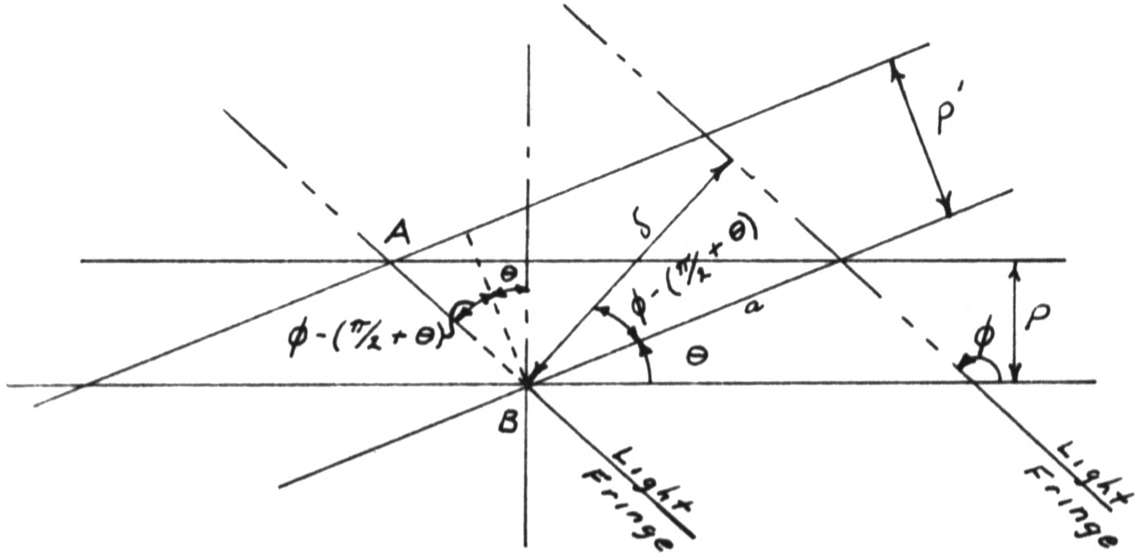


FIGURE (1)

Geometry of Moire' Fringes

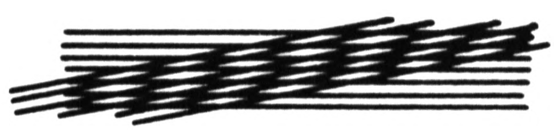


FIGURE (2)

Formation of Moire' Fringes

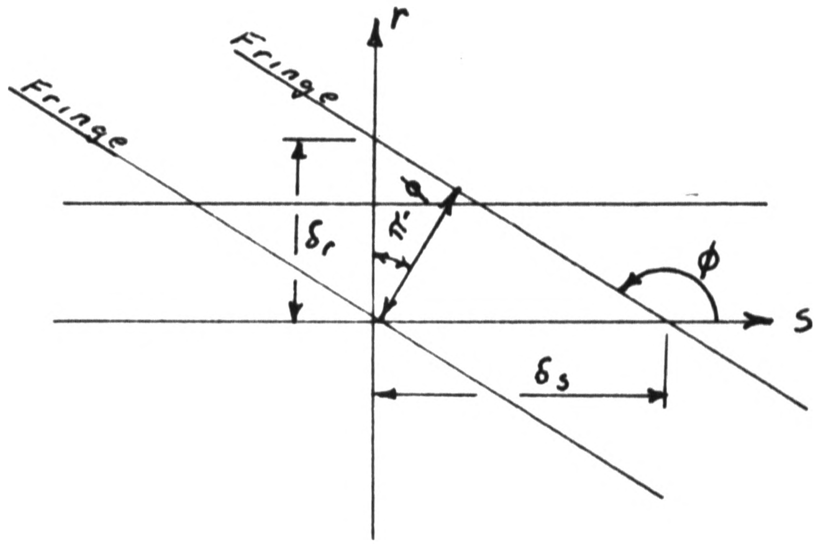


FIGURE (3)

Geometry of Fringe Measurements in Coordinate Directions

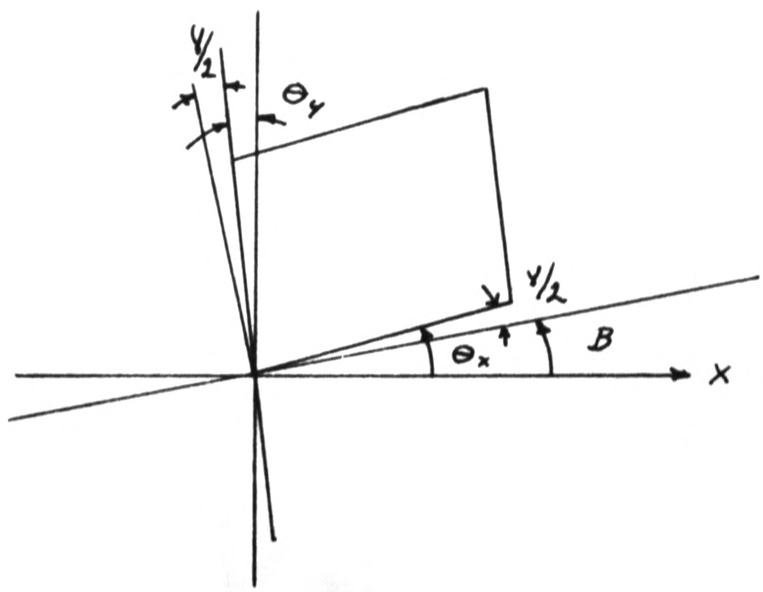
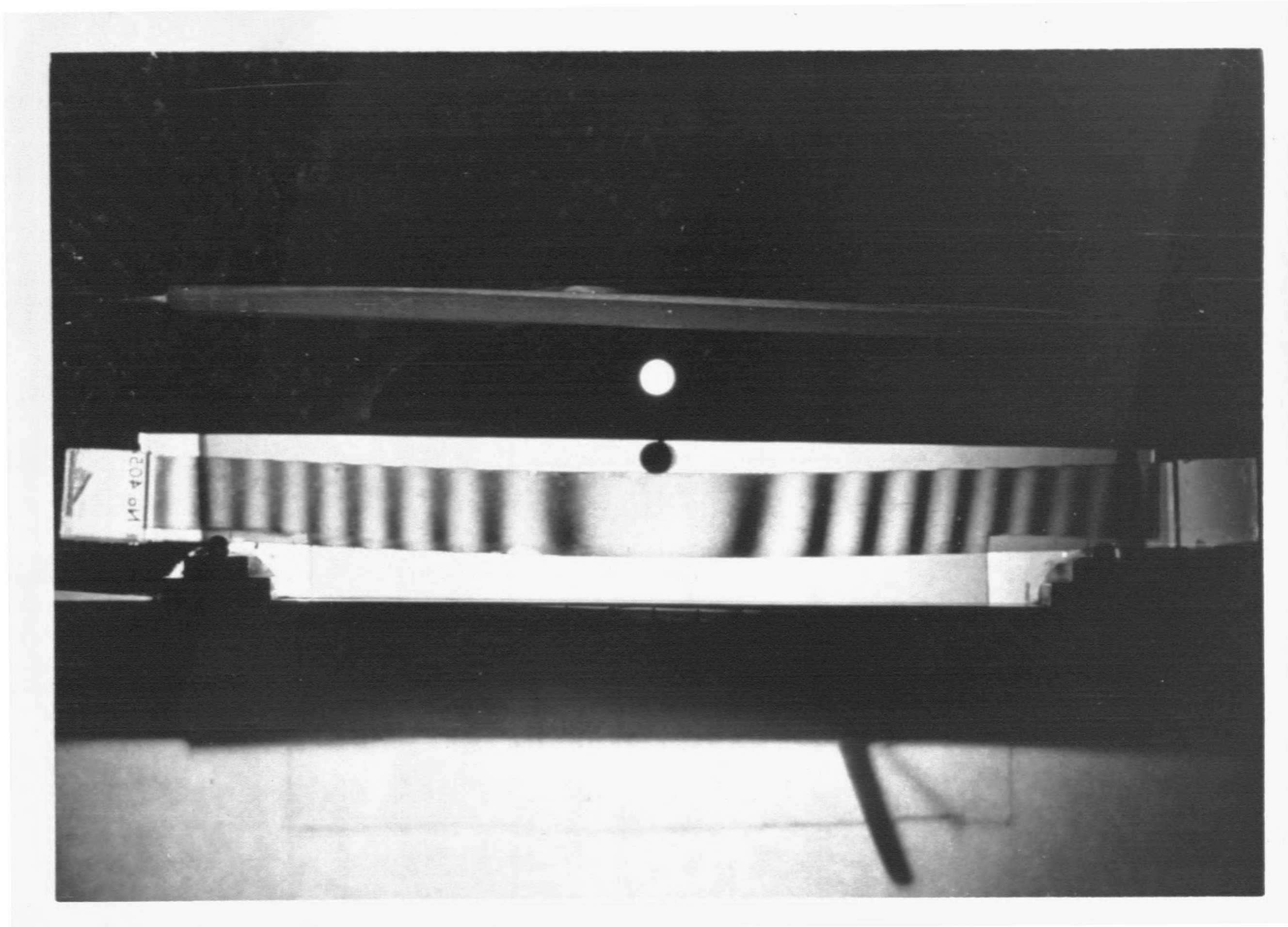


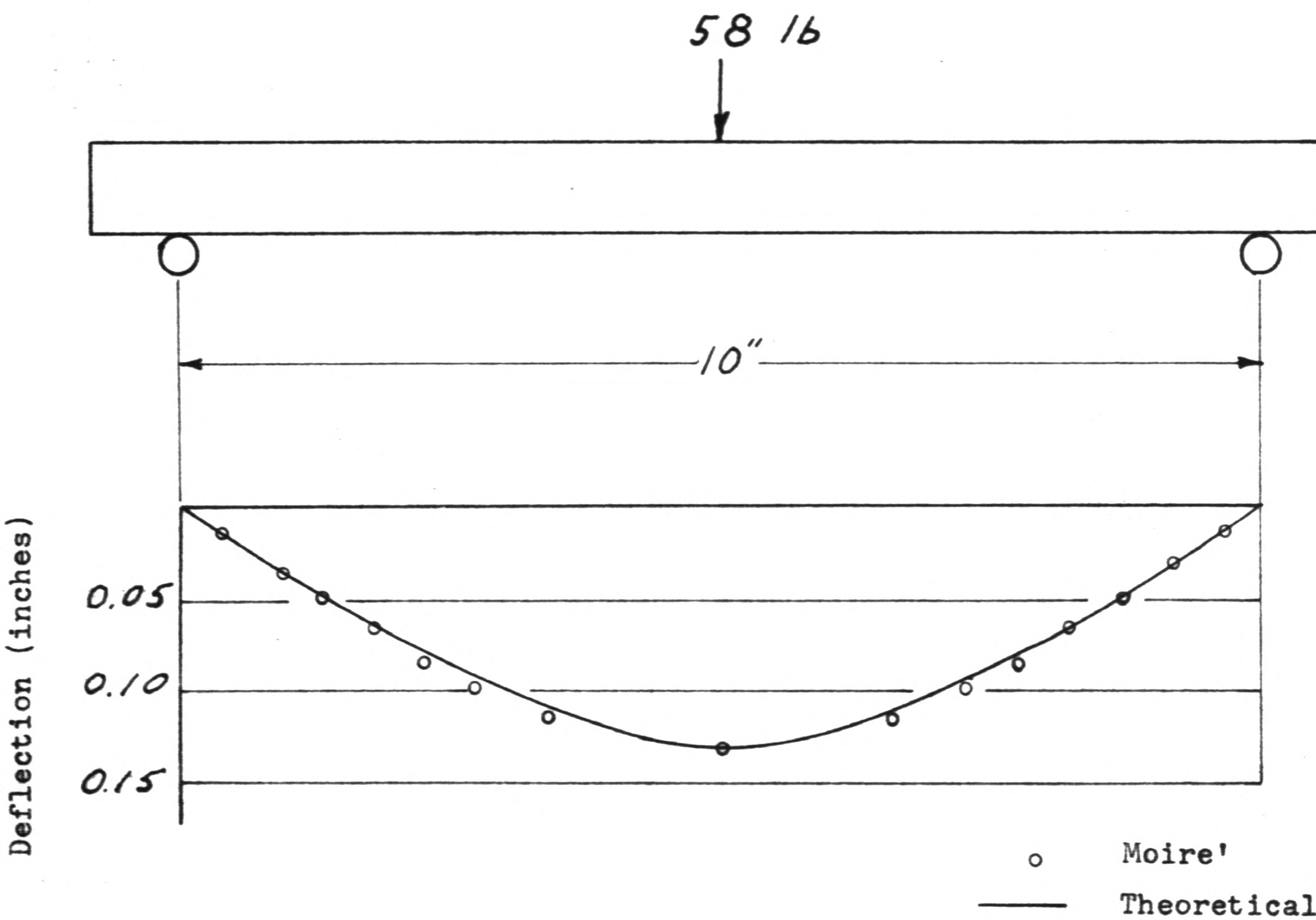
FIGURE (4)

Geometry of Shear Strain and Rigid Body Rotation



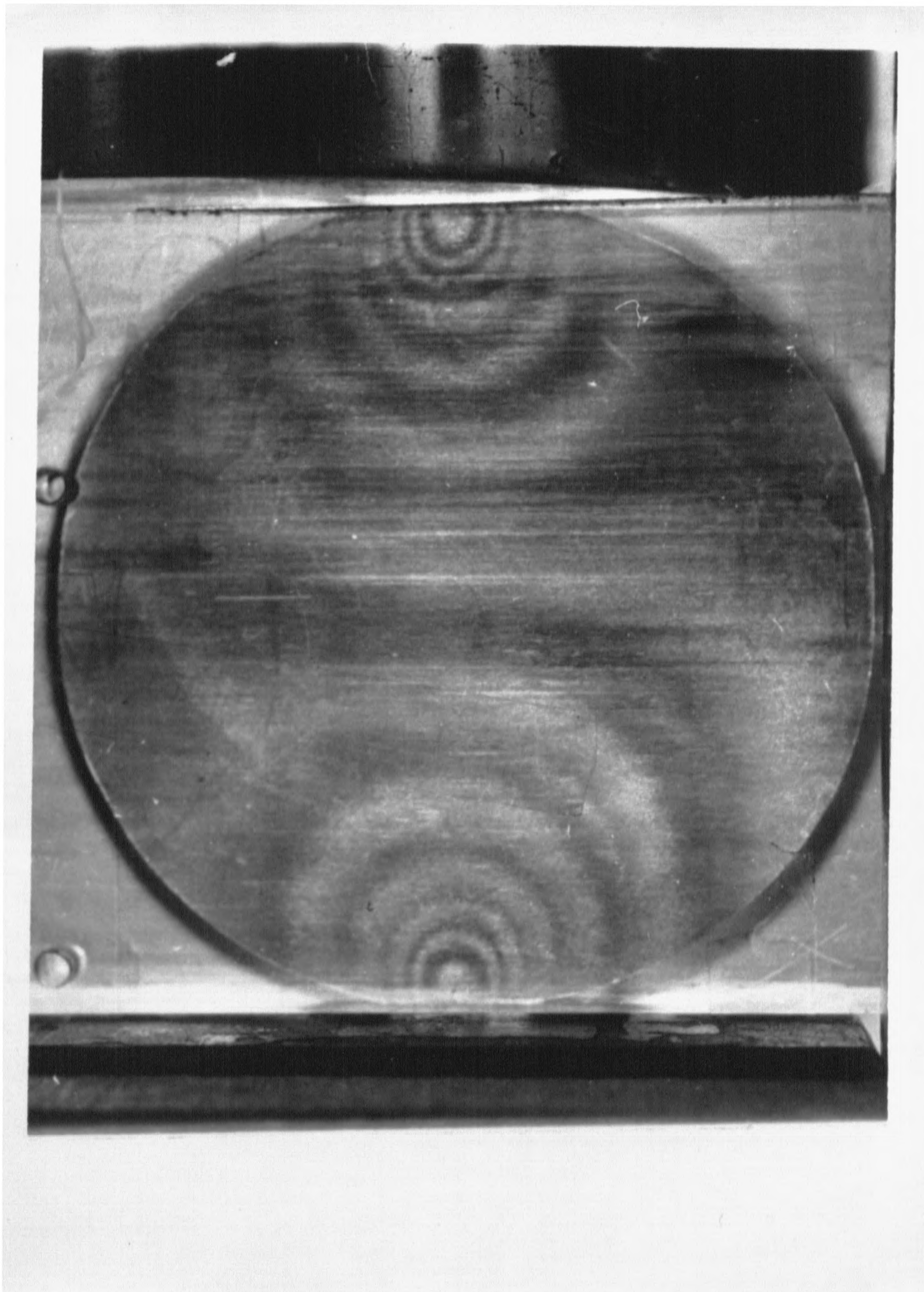
Beam Deflection Test with Horizontal Grid of 60 Lines per Inch. 58 Pound Load at Center of 10 Inch Span.

Figure 5



Deflection Curve of Beam

FIGURE (6)



Diammetrically Loaded Disk with 200 Lines per Inch Grid Perpendicular to 4000 Pound Load.

Figure 7

ARBITRARY FRINGE NUMBER

VERSUS

ACCUMULATED FRINGE DISTANCE

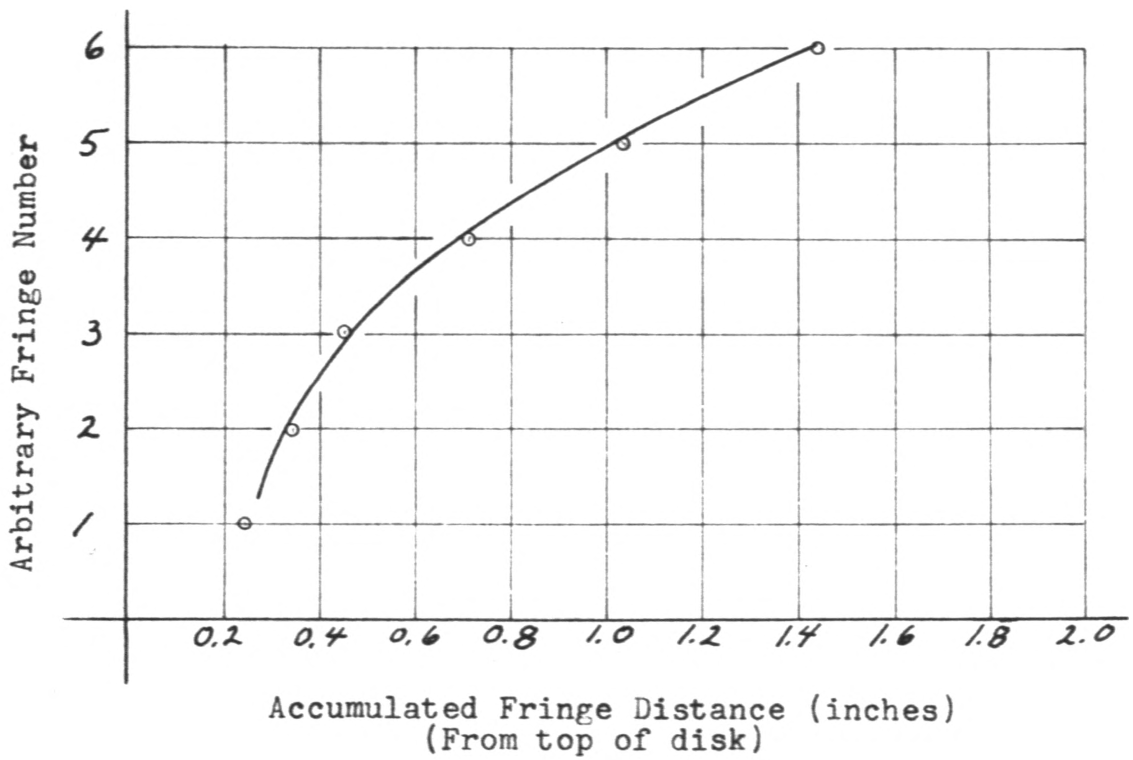
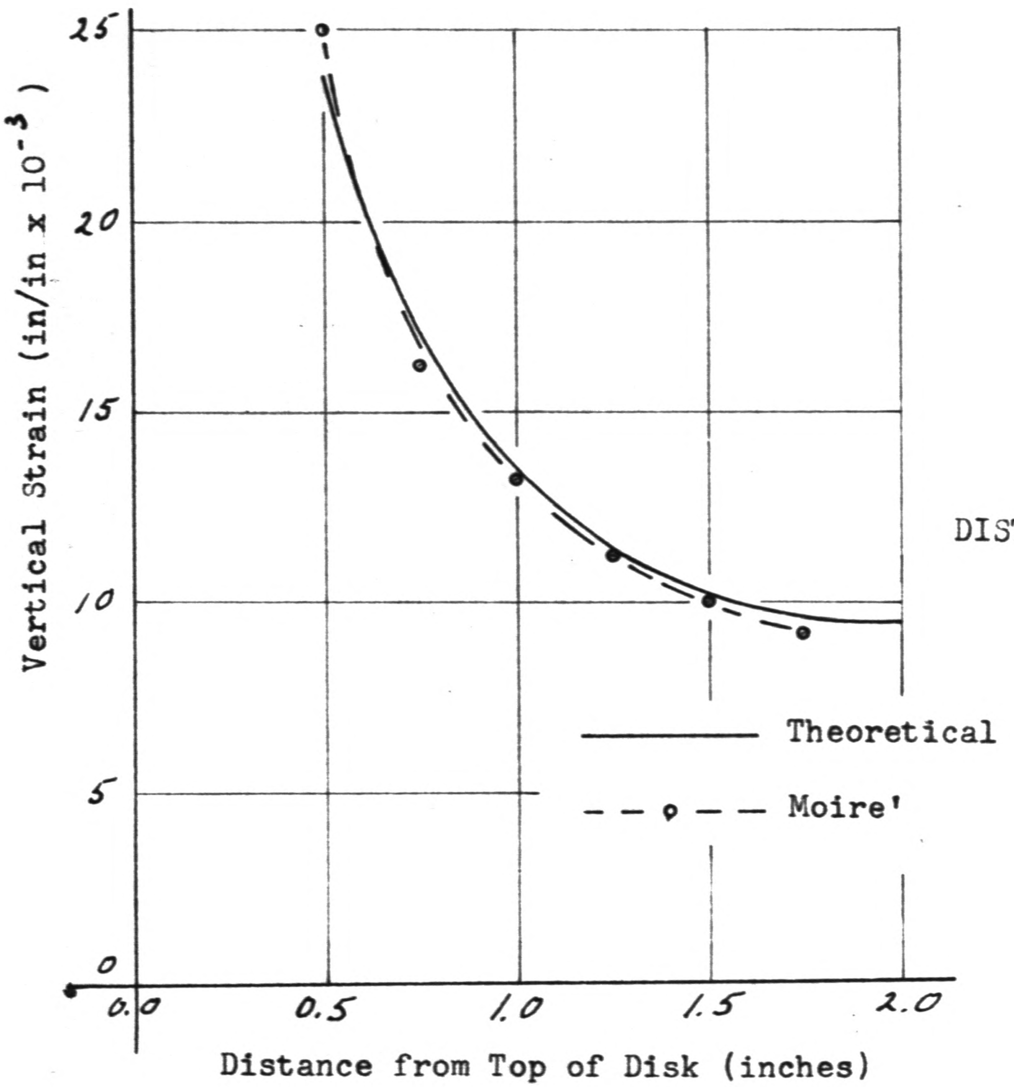


FIGURE (8)

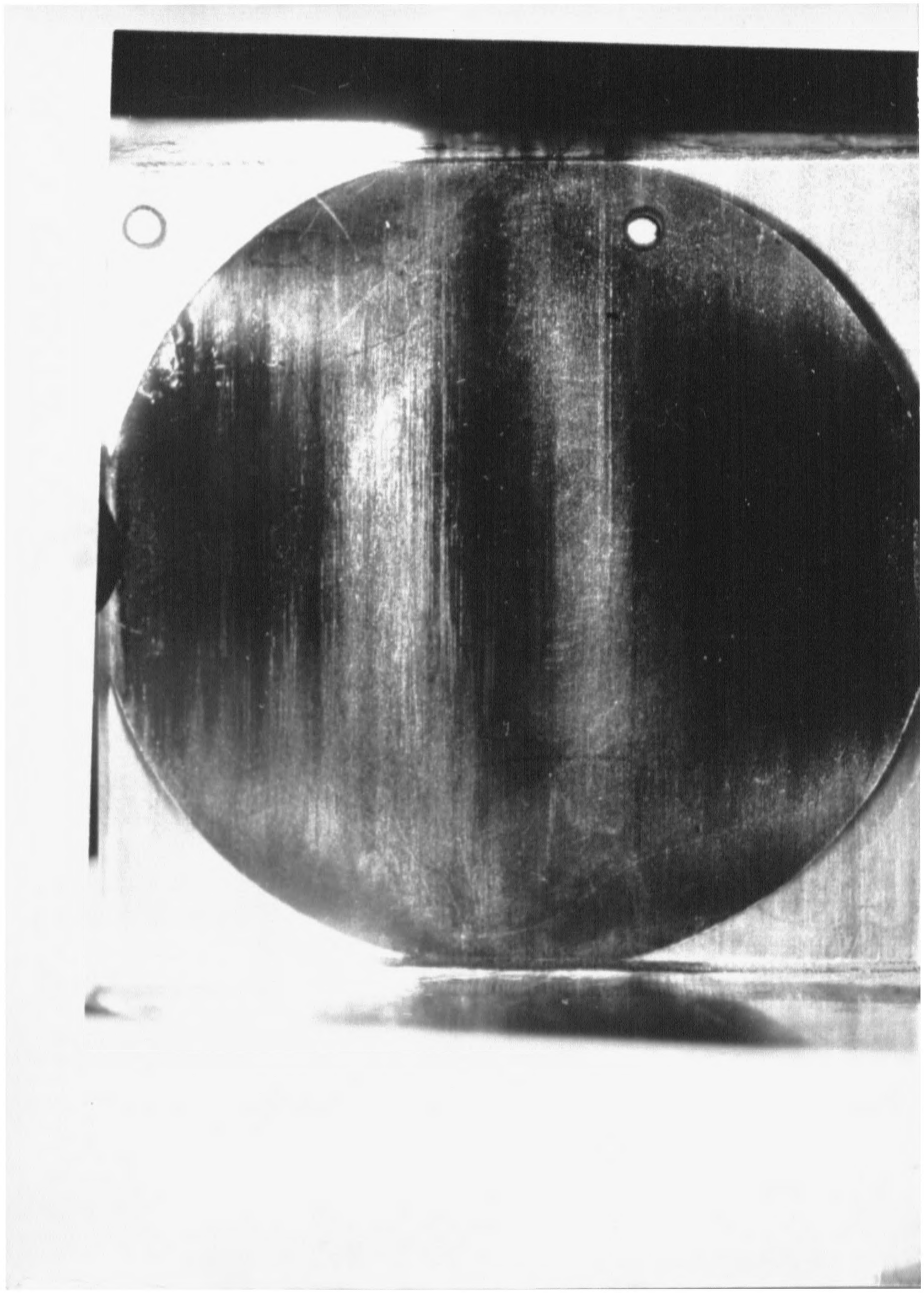


VERTICAL STRAIN

VERSUS

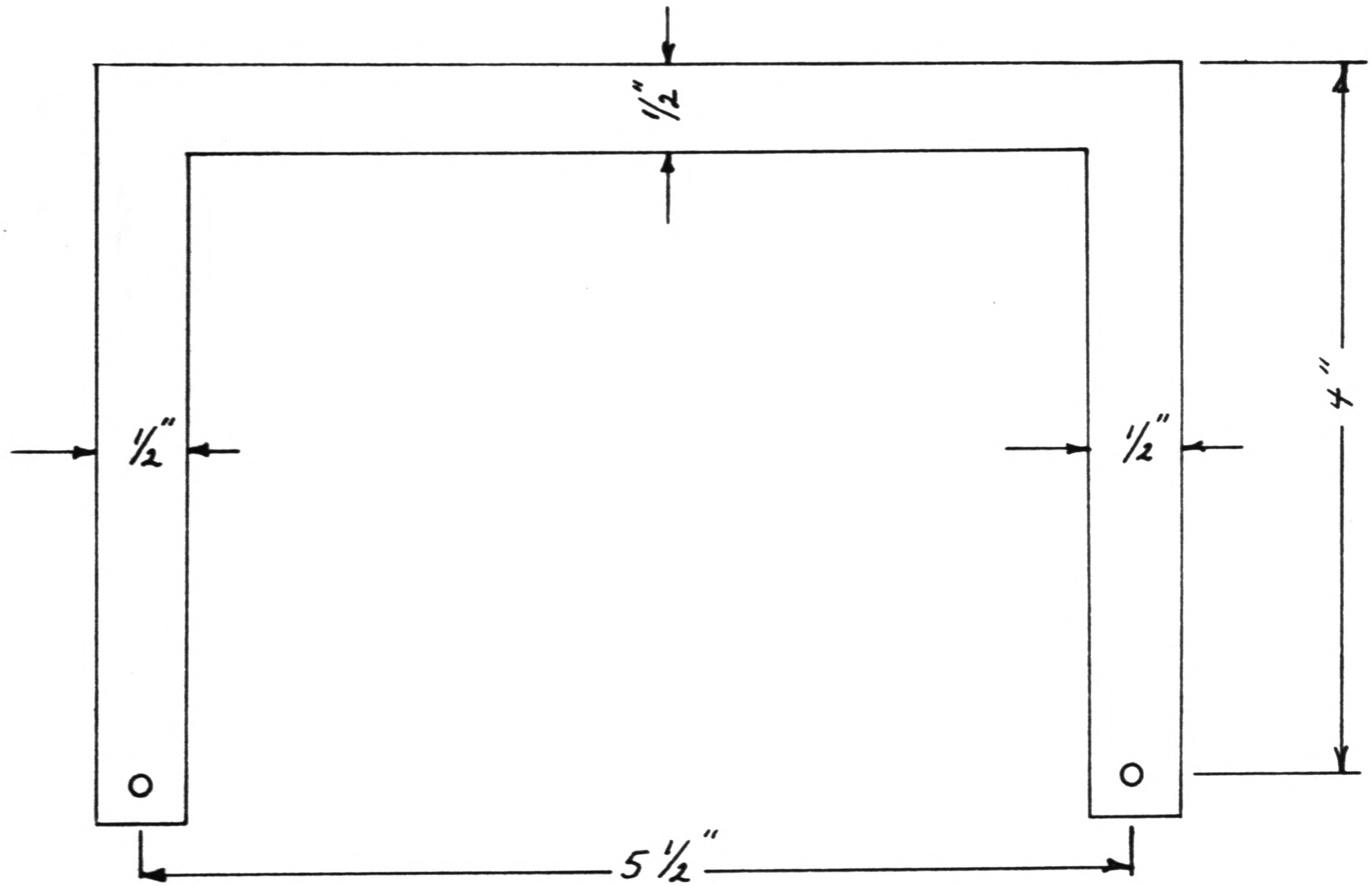
DISTANCE FROM TOP OF DISK

FIGURE (9)



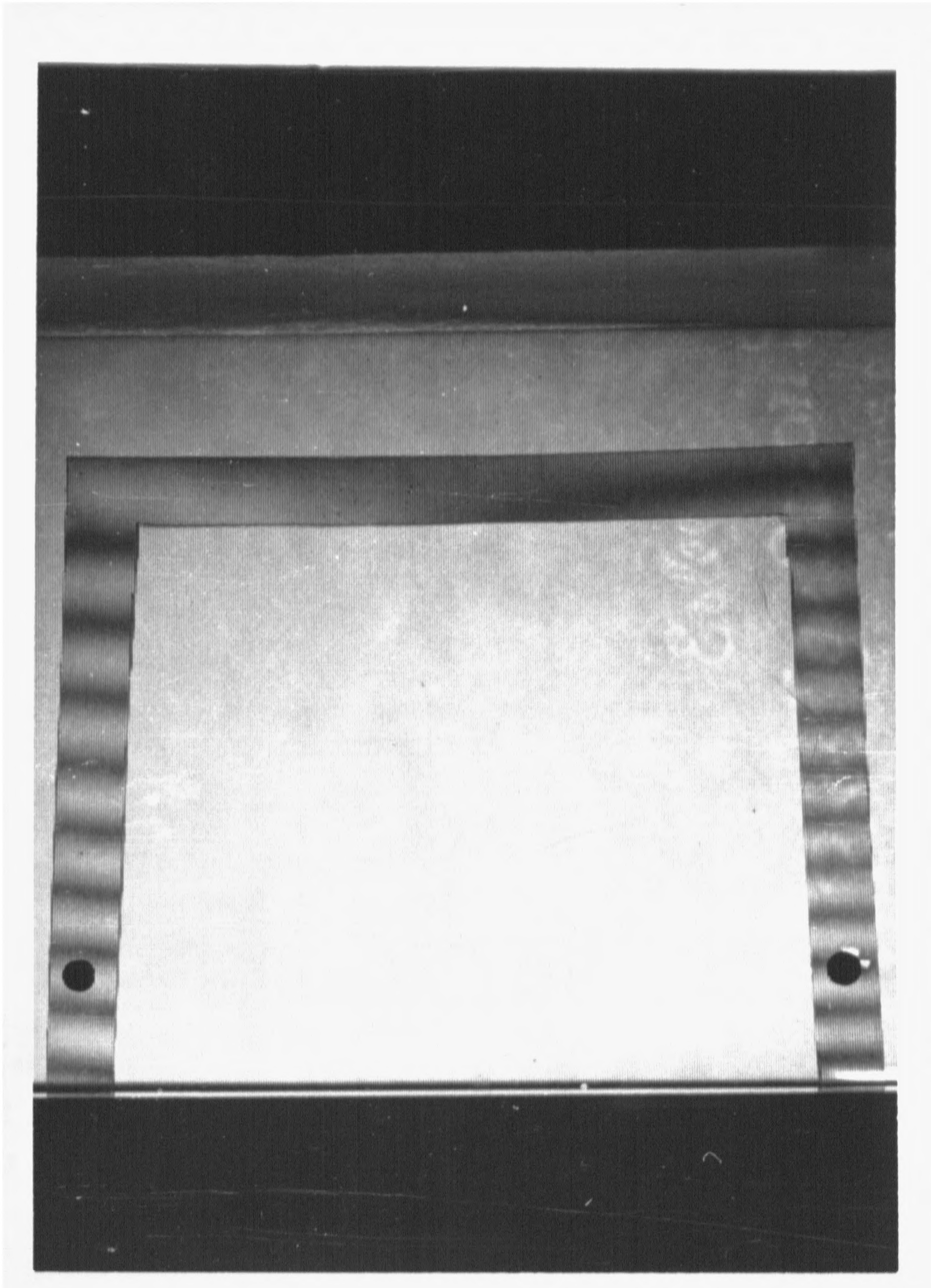
Diammetrically Loaded Disk with 200 Lines per Inch Grid Parallel to 4000 Pound Load.

Figure 10



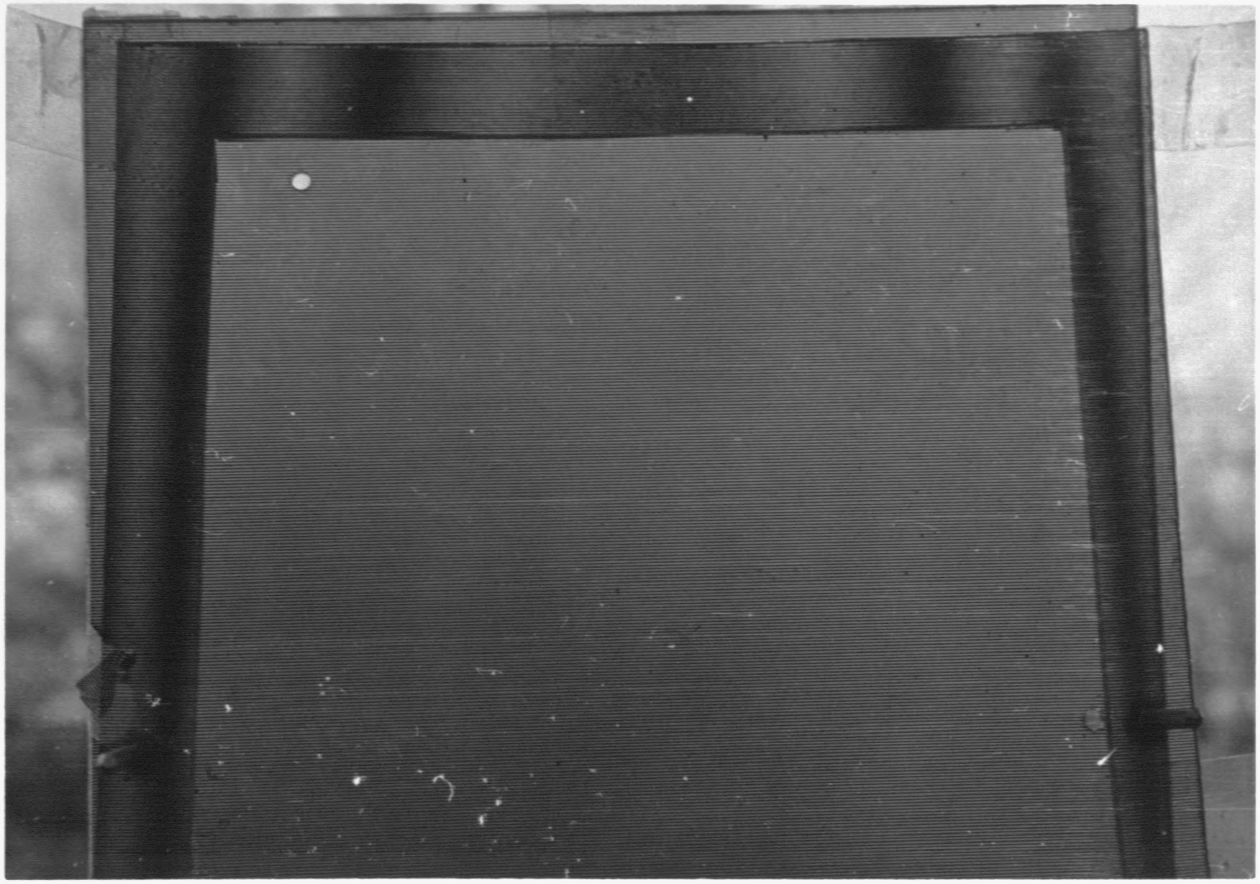
Dimensions of Frame Tested

FIGURE (11)



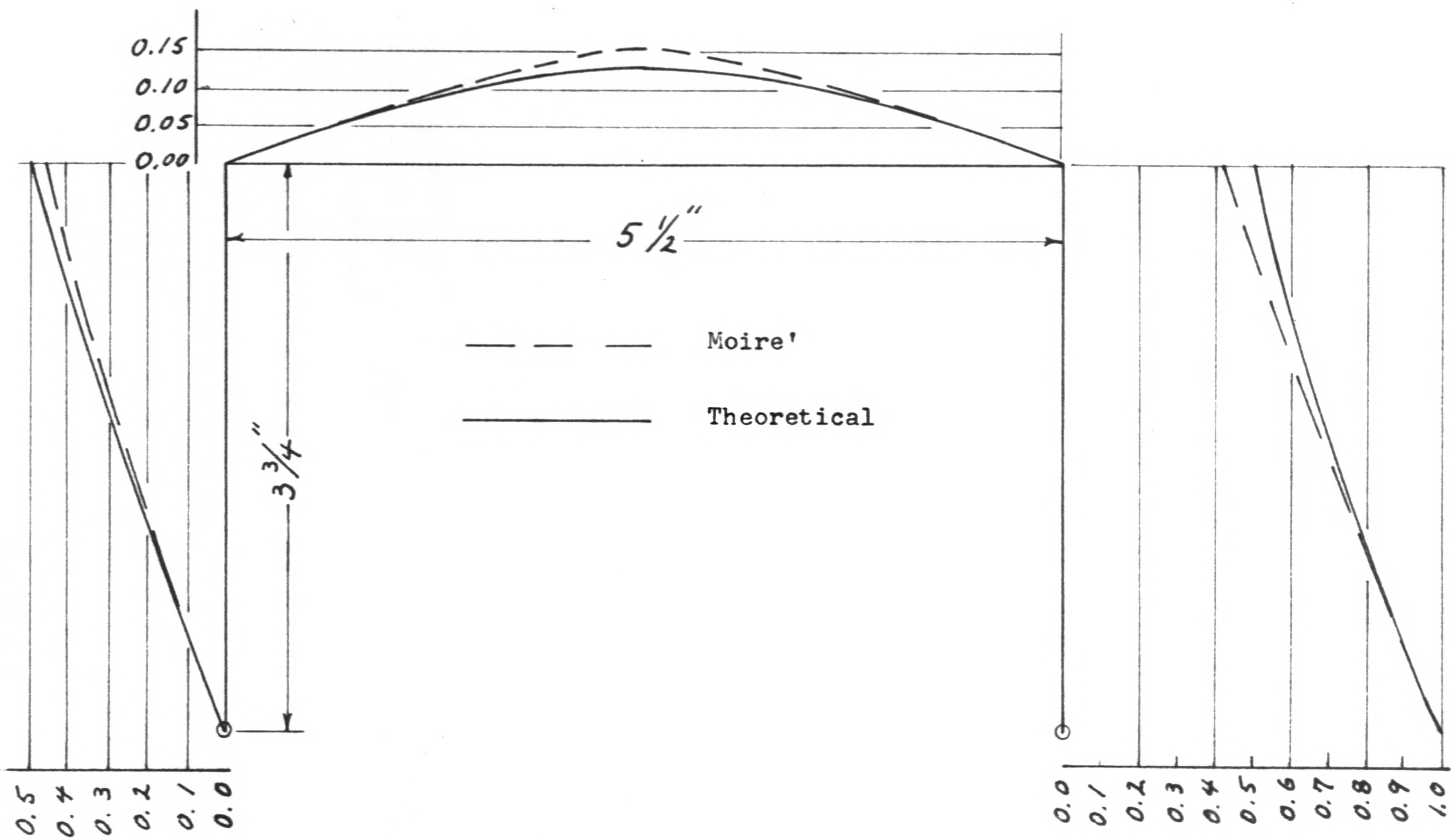
Frame Deflection Test with Vertical Grid of 60 Lines per Inch. Right Support Displaced 0.26 inches Horizontally.

Figure 12



Frame Deflection Test with Horizontal Grid of 60 Lines per Inch.
Right Support Displaced 0.26 Inches Horizontally.

Figure 13



Influence Diagram for Right
Horizontal Reaction of Frame

FIGURE (14)

RESULTS AND CONCLUSIONS

Throughout this study two uses of Moire' fringes have been considered: the measurement of deflections, and the measurement of strains. These will be discussed separately as follows:

Deflections:

1) The Moire' method, from this study and from Literature, seems to be an accurate, economical, and time saving method of obtaining deflections. The Begg's Deformeter is more accurate in the measurement of deflections but has the disadvantages of being slower and it does not give a composite picture of deflection at any stage of loading as in the Moire' Method.

2) The use of Moire' fringes for the measurement of deflections as well as other problems, depends on the proper selection and application of a grid. In this study, commercially available clear plastic sheets which are ruled and have a pressure sensitive cement backing were used. These sheets with rulings up to 60 lines per inch are readily available at artist supply stores under the names of Artype and Zip-A-Tone. No difficulties were encountered in placing these grids on the models. Using a grid of 60 lines per inch each clear fringe represents a deflection of 0.0167 inches from the preceding clear fringe.

3) The stationary or master grid must be held against the model grid and oriented before testing so that no fringes can be seen. It was found that if the two grids were separated, even slightly, the fringes became hazy. This is due to the light bending around the lines in the grid thus losing the shadow effect explained earlier. As an example of this, in the beam test if the two grids were separated by the thickness of the beam, 1/4 of an inch, no fringes could be seen.

4) The use of photographs seems to be the best method of analyzing the fringes. There was no trouble in obtaining photographs with clear fringes since a white light source was placed behind the grids to bring out the fringes. The fringes should be observed from directly in front of the grids since if they are viewed from an angle, the fringes would be slightly displaced from their correct position. If the model was very large, perhaps several pictures would have to be taken from different positions in order to determine the correct location of the fringe.

Strains:

1) The Moire' method provides a means by which large strains can be measured. Since most of the existing strain instruments are inadequate for measuring large strains, this could prove to be an important use of Moire' fringes.

2) As in the case of deflection measurements, the accurate measurement of strains depends on the proper selection and use of a grid. In this study a grid of 200 lines per inch was machined on the model mechanically by cutting grooves in the material and then darkening these grooves. Perhaps a better method, if the equipment is available, is to photoprint the grid on the specimen. In any case a uniform grid of probably not less than 200 lines per inch would be necessary for the measurement of strains. By looking at the approximate equation for strain, it can be seen that for any given amount of strain, the distance between the fringes is proportional to the grid spacing. Since it is necessary to measure the distance between the fringes, the smaller the grid pitch, the more accurate would be the measurement of the distance between the fringes and the more accurate would be the strain readings. Assuming that the grid pitch was 300 lines per inch and that the maximum distance between the fringes that could accurately be measured was 1/2 of an inch, the least strain that could be measured would be 0.0066 inches per inch. This is a relatively large amount of strain and would be in the plastic range of many materials.

3) The master grid must be against the model grid, as before, in order to see the fringes. Because of the finer grids, orientation is more of a problem here than in the measurement of deflections. The method used in this study was to

put an initial load on the specimen and then knowing that the strain in the disk would be symmetrical about the horizontal and vertical diameters, the master grid was oriented such that the fringes were symmetrical with respect to these two axis. Because of rather crude equipment this was not completely accomplished. By looking at figure 7 it can be seen that the fringes are symmetrical with respect to the vertical diameter, but the center four fringes are not symmetrical with respect to the horizontal diameter. In calculating the strain from these fringes good results were obtained by adjusting their positions to make them symmetrical. This method, of course, would not work in all problems. A frame which would hold the master grid and could be rotated with a fine adjustment would be helpful in overcoming this problem.

4) Because of the fineness of the lines in smaller pitch grids, the fringes become faint making photography more of a problem. The pictures in this study were taken with a 35 mm camera equipped with a Zeis number 3 lens. The pictures were taken from approximately 10 inches. Illumination was provided with a 200 watt lamp about 2 feet behind the model. The method of re-exposing the film described in the review of literature would eliminate some of the problems of orientation but would create new problems in photography. In this method the camera and film must be

capable of clearly resolving the grid on the negative through two exposures of the film.

5) In order to measure the principal strains in a model two grids would have to be put on the model in two perpendicular directions and pictures taken of the fringes with the grids oriented in the two directions. These two pictures along with the previously developed equations for the normal strain and the shear strain would be sufficient to determine the principal strains at any point.

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